Inference at $*$ 1 0 1 1 1
 of proof for Lemma integer sqrt:

1. $n : \mathbb{Z}$
2. $n < 0$
3. $((n{+}1) \geq 0\ ) \Rightarrow (\exists r{:}\mathbb{N}.\ (((r * r) \leq (n{+}1))\ \&\ ((n{+}1) < ((r{+}1) * (r{+}1)))))$
4. $n \geq 0$

$\vdash \exists r{:}\mathbb{N}.\ (((r * r) \leq n)\ \&\ (n < ((r{+}1) * (r{+}1))))$
  by Arith

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http://www.nuprl.org/FDLcontent/p0_963683_/p28_1027382_{integer!sqrt}1_0_1_1_1.html